Classification of Constraints:

A Constraint is:

(i) (a) Scleronomic: If Constraint relations do not explicity depend on time. in Case of a rigid body.

(b) Rheonomic: If Constraint relations depend explicity on time. e.g. a bead sliding on a moving wire

(ii) (a) Holonomic: If constraint relations are or can be made independent of velocity e.g. a cylinder rolling without sliding down an inclined plane.

(b) Non-holonomic: If Constraint relations are not holonomic e.g. a sphere rolling without sliding down an inclined plane.

(iii) (a) Bilateral: If the Constraint relations are in the form of equations e.g. rigid body.

(b) Unilateral: If the Constraint relations are expressed in the form of inequalities e.g. motion

of molecules in a gas container.

(iv) (a) Conservative : If forces of constraint do not do any work and total mechanical energy of the system is conserved e.g. simple pendulum with rigid support.

(b) Dissipative : If the forces of constraint do work and the total mechanical energy is not conserved e.g. pendulum with variable length.

Constraints in some cases:

System	Type of Constraint
1. Rigid body	Scleronomic, holonomic, bilateral Conservative
2. Deformable bodies	Rheonomic, holonomic, bilateral dissipative
3. Simple pendulum with rigid support	Scleronomic, holonomic, bilateral, Conservative
4. Pedulum with variable length	Rheonomic, holonomic, bilateral, dissipative

Degrees of freedom:

The number of independent ways in which a mechanical system can move without violating any constraint which may be imposed, is called the number of degrees of freedom of the system. It is indicated by the least possible number of co-ordinates to describe the system completely.

For ex.:- When a single particle moves in space, it has three degrees of freedom, but if it is constrained to move along a certain space curve, it has only one.

In a system of N particles subjected to k independent constraints expressible in k equations of the form

$$\left.\begin{aligned} g_1(x_1, y_1, \cdots z_N, t) &= a_1 \\ g_2(x_1, y_1, \cdots z_N, t) &= a_2 \\ \cdots\cdots\cdots\cdots\cdots & \\ g_k(x_1, y_1, \cdots z_N, t) &= a_k \end{aligned}\right\} \text{---} (1)$$

the number of degrees of freedom, f are $3N-k$; that is

$$f = 3N - k \quad \text{——} (2)$$

Here ① $q_1, q_2, \ldots q_k$ are k specified functions of $3N$ co-ordinates and possibly of time if constraints depend on time explicitly.